CSR AND MICROBUNCHING IN BUNCH COMPRESSORS

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Outline

VM3@A (Vlasov Maxwell Monte-carlo Method at Albuquerque)

- Self-consistent 2D Vlasov-Maxwell solver for accurate modeling of CSR effects and microbunching instability in bunch compressors
- Monte-carlo particle method implemented to solve the Vlasov-Maxwell system with a 2D (z-x) mean field approximation
- Equations of motion solved in the interaction picture defined by the principal solution matrix of the linearized equations of motion without self fields
- Field calculation based on a Green's function method with integration over 2D "history" of the charge/current density
- Vacuum chamber modeled with perfect conducting parallel plates
- 1D field approximation scheme for rapid simulations
- References

G. Bassi, J.A. Ellison, K. Heinemann and R. Warnock, PRSTAB **12**, 080704 (2009)
G. Bassi, J.A. Ellison, K. Heinemann and R. Warnock, PRSTAB **13**, 104403 (2010)
B. Terzic and G. Bassi, PRSTAB **14**, 070701 (2011))

FERMI@Elettra BC1: chicane and beam parameters at first dipole

Parameter	Symbol	Value	Unit
Energy reference particle	E_r	285	MeV
Peak current	I	55	A
Bunch charge	Q	0.5	nC
Norm. transverse emittance	$\gamma\epsilon_0$	1.5	μm
Alpha function	α_0	3.5	
Beta function	β_0	40	m
Linear energy chirp	h	21.54	1/m
Uncorrelated energy spread	σ_E	10	KeV
Momentum compaction	R_{56}	-41.8	mm
Magnetic length	L_b	0.366	m
Distance 1st-2nd, 3rd-4th bend	L_1	2.6468	m
Distance 2rd-3nd bend	L_2	1.1833	m

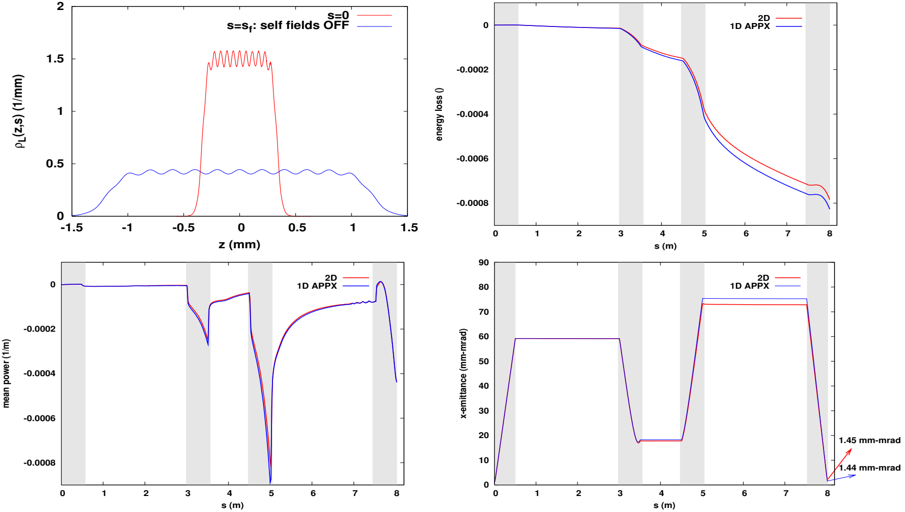
Comparison (preliminary) VM3@A vs Elegant
(Elegant simulations by S. Di Mitri)

Parameter	Elegant	VM3@A
σ_x initial (x-band exit in Elegant), μm	291.7	298
σ_x final, μm	97.3	98
$\sigma_{x'}$ initial, μ-mrad	29.7	29.7
$\sigma_{x'}$ final, μ-mrad	31.1	39.8
σ_δ initial	0.018	0.018
σ_δ final	0.018	0.018
ϵ_x projected and normalized, initial, μ-mrad	1.459	1.503
ϵ_x projected and normalized, final, μ-mrad	1.540	1.529
Bunch length, initial	849	851
Bunch length, final	87	91

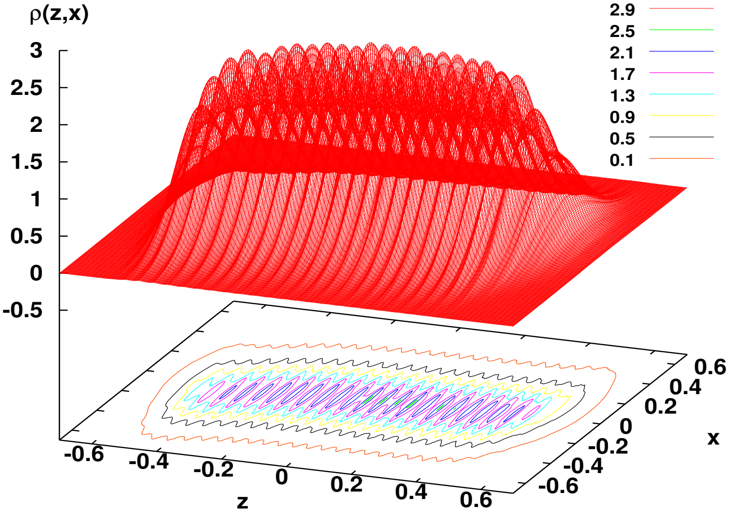
FERMI@Elettra BC: chicane and beam parameters at first dipole
(Benchmark parameters for code comparison (from μBI workshop 2007))

Parameter	Symbol	Value	Unit
Energy reference particle	E_r	233	MeV
Peak current	I	120	A
Bunch charge	Q	1	nC
Norm. transverse emittance	$\gamma\epsilon_0$	1	μm
Alpha function	α_0	0	
Beta function	β_0	10	m
Linear energy chirp	h	-27.5	1/m
Uncorrelated energy spread	σ_E	2	KeV
Momentum compaction	R_{56}	0.025	m
Radius of curvature	ρ_0	5	m
Magnetic length	L_b	0.5	m
Distance 1st-2nd, 3rd-4th bend	L_1	2.5	m
Distance 2rd-3nd bend	L_2	1	m

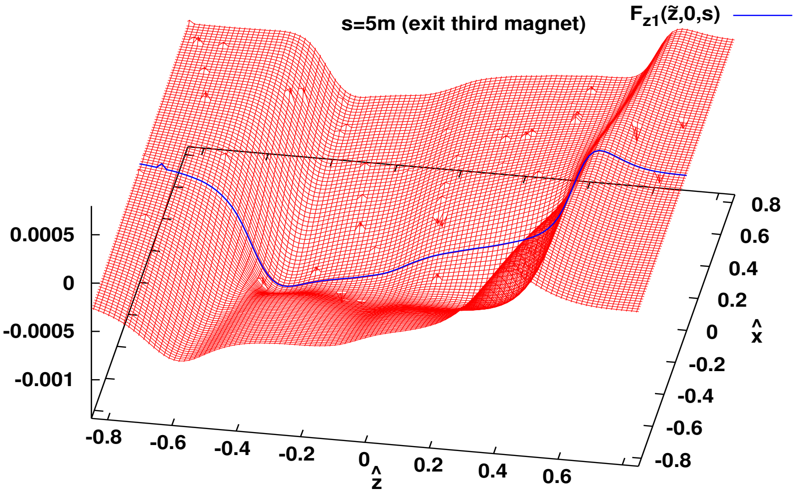
Microbunching instability simulations in the FERMI@Elettra BC



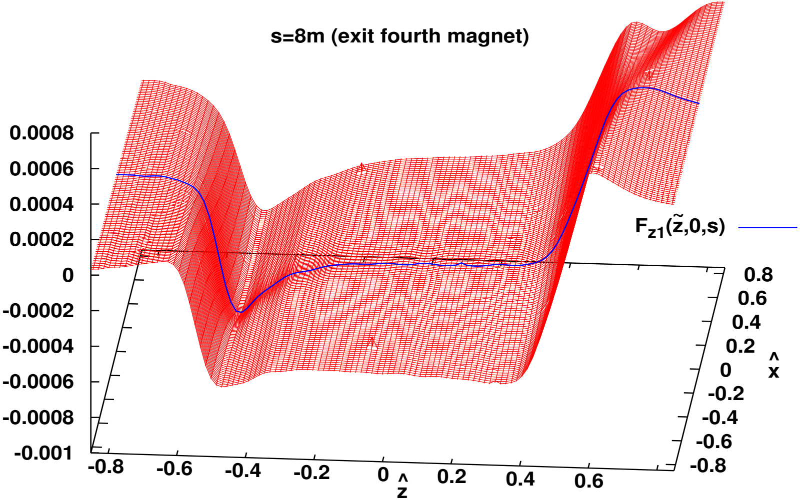
2D spatial density at BC exit for $\lambda = 100\mu$m



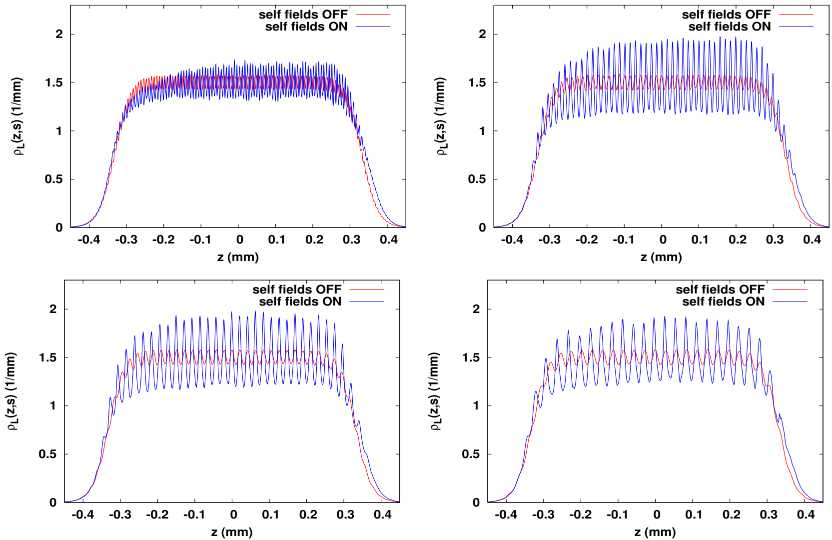
Comparison 2D vs 1D field calculation



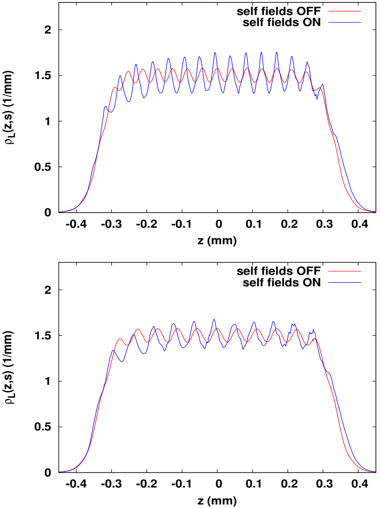
Comparison 2D vs 1D field calculation



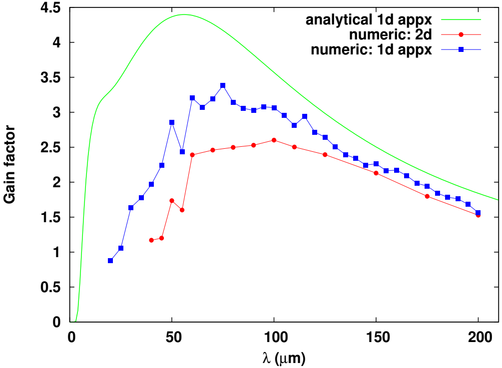
Longitudinal spatial density at BC exit for $\lambda = 25, 50, 75, 100\mu$m



Comparison gain factor



$\lambda = 150\mu$m (top), $\lambda = 200\mu$m (bottom)



Energy-to-Density Modulation Conversion in a Bunch Compressor
(E. Saldin, E. Schneidmiller and M. Yurkov, NIMA 483, 516 (2002))

Let us assume a coasting beam for the longitudinal current density with phase space density (PSD) $f_0(z,\delta)$ at the entrance of the BC

$$f_0(z,\delta) = \frac{I_0}{\sqrt{2\pi}\sigma_u}\exp\Big(-\frac{[\delta - hz + \Delta\delta\sin kz]^2}{2\sigma_u^2}\Big),$$

where $h = (1-C)/(CR_{56})$ is the linear chirp and $\Delta\delta$ the amplitude of a sinusoidal energy modulation with wavelength $\lambda = 2\pi/k$.

The PSD $f(z,\delta)$ and current density $I(z) = \int d\delta f(z,\delta)$ at BC exit read

$$\begin{aligned} f(z,\delta) &= \frac{I_0}{\sqrt{2\pi}\sigma_u}\exp\Big(-\frac{[\delta - h(z-R_{56}\delta)) + \Delta\delta\sin k(z-R_{56}\delta)]^2}{2\sigma_u^2}\Big), \\ I(z) &= CI_0\Big[1 + 2\sum_{n=1}^{\infty} J_n\Big(nCkR_{56}\Delta\delta\Big)e^{-\frac{1}{2}n^2C^2k^2R_{56}^2\sigma_u^2}\cos(nCkz)\Big], \end{aligned} \quad (1)$$

where J_n is the Bessel function of nth order.

Using the approximation $A = Ck|R_{56}|\Delta\delta \ll 1$, $J_1(A) \approx A/2$

$$I(z) \approx CI_0\Big[1 + CkR_{56}\Delta\delta e^{-\frac{1}{2}C^2k^2R_{56}^2\sigma_u^2}\cos(Ckz)\Big]. \quad (2)$$

Gain Factor Formula

The approximate Eq.(2) is used to define the gain factor

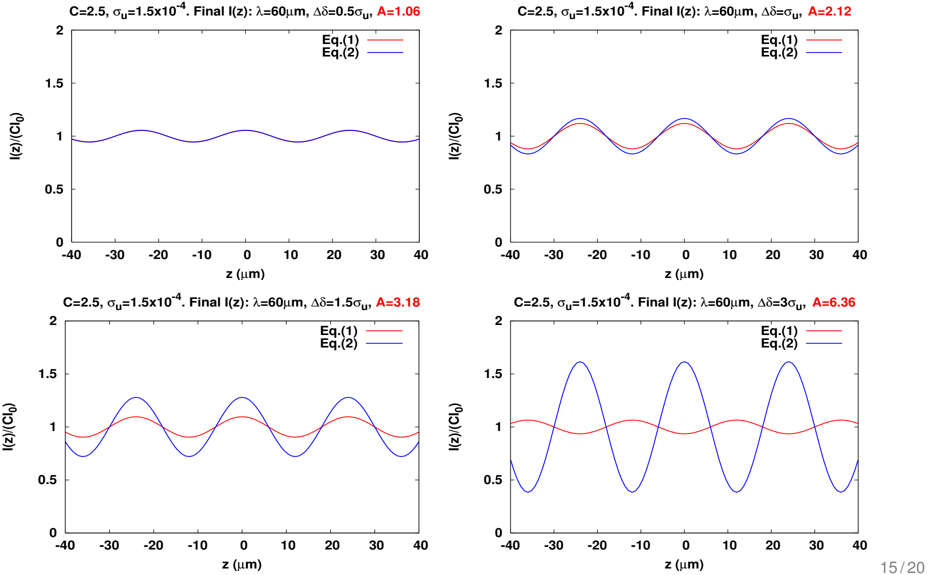
$$G = \frac{\rho_{\text{ind}}}{\rho_i} = Ck|R_{56}|\frac{\Delta\delta}{\rho_i}e^{-\frac{1}{2}C^2k^2R_{56}^2\sigma_u^2}, \quad (3)$$

for an initial current modulation with amplitude ρ_i and wave number $k = 2\pi/\lambda$.

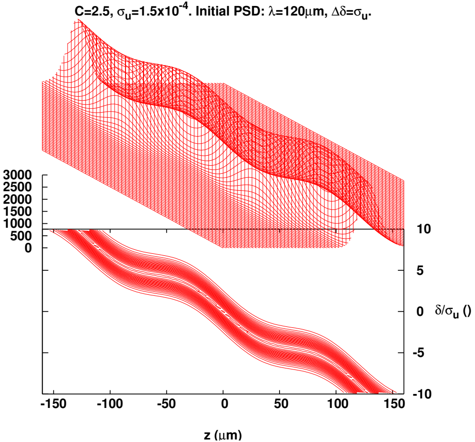
Eq.(3) holds for a sinusoidal energy modulation $\sin kz$ with amplitude $\Delta\delta$ and a coasting beam assumption at the entrance of the BC, in the limit $\rho_i \ll \rho_{\text{ind}} \ll 1$.

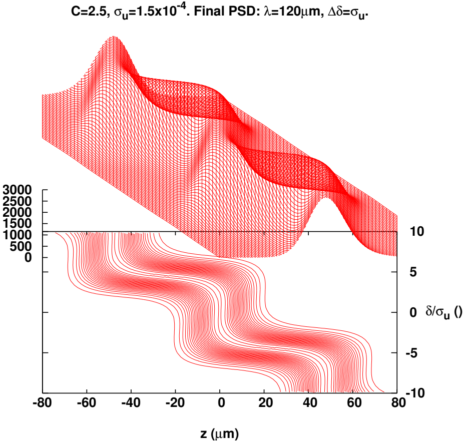
Eq.(3) has been used by Seletskiy et al. (PRL 111, 034803 (2013)) outside its range of validity (violating the condition A$\ll 1$), nevertheless obtaining a good fit with the experimental gain factor. See talk of Xi Yang on Tuesday.

Longitudinal Current Density at BC Exit for $\lambda = 60\mu$m

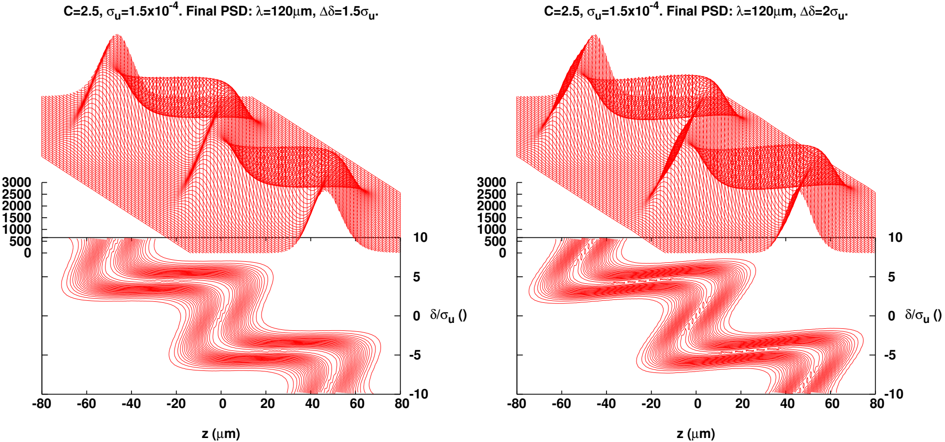


Longitudinal Phase Space Density for $\lambda = 120\mu$m





Longitudinal Phase Space Density at BC Exit for $\lambda = 120\mu$m



Proof of the Energy-to-Density Modulation Conversion Formula

$$I(z) = \frac{I_0}{\sqrt{2\pi}\sigma_u}\int_{-\infty}^{\infty} d\delta \exp\left(-\frac{[\delta - h(z - R_{56}\delta) + \Delta\delta \sin k(z - R_{56}\delta)]^2}{2\sigma_u^2}\right),$$
$$= A_0 \int_{-\infty}^{+\infty} dx \exp\left(-\frac{[x + \Delta\delta \sin(Ckz - CkR_{56}x)]^2}{2\sigma_u^2}\right), \quad (4)$$

where $A_0 = CI_0/(\sqrt{2\pi}\sigma_u)$ and we made the change of variable $x = \delta - h(z - R_{56}\delta)$. Since $I(z)$ is even in z [1] and periodic with period $T = 2\pi/(Ck)$, we expand it in cosine Fourier series

$$I(z) = I(-z), \quad I(z) = I\left(z + \frac{2\pi}{Ck}\right) \Longrightarrow I(z) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nCkz)$$

$$a_n = \frac{Ck}{\pi}\int_{-\frac{\pi}{Ck}}^{\frac{\pi}{Ck}} dz\, I(z)\cos(nCkz). \quad (5)$$

Thus, with $B = CkR_{56}$ we have

$$a_n = \frac{A_0 Ck}{\pi}\int_{-\frac{\pi}{Ck}}^{\frac{\pi}{Ck}} dz \int_{-\infty}^{+\infty} dx \exp\left(-\frac{[x + \Delta\delta \sin(Ckz - Bx)]^2}{2\sigma_u^2}\right)\cos(nCkz). \quad (6)$$

[1] $I(-z) = A_0 \int_{-\infty}^{+\infty} dx\ \exp[-(x - \Delta\delta \sin(kCz + Bx))^2/(2\sigma_u^2)] \overset{x'=-x}{=} -A_0 \int_{+\infty}^{-\infty} dx'\ \exp[-(-x' - \Delta\delta \sin(kCz - Bx'))^2/(2\sigma_u^2)] = A_0 \int_{-\infty}^{+\infty} dx'\ \exp[-(x' + \Delta\delta \sin(kCz - Bx'))^2/(2\sigma_u^2)] = I(z)$, where $B = CkR_{56}$.

Proof of the Energy-to-Density Modulation Conversion Formula

Changing variable $y = Ckz - Bx, dz = dy/(Ck)$ we have

$$\begin{aligned} a_n &= \frac{A_0}{\pi}\int_{-\infty}^{+\infty} dx \int_{-\pi-Bx}^{\pi-Bx} dy \, \exp\left(-\frac{[x+\Delta\delta \sin y]^2}{2\sigma_u^2}\right)\cos n(y+Bx) \\ &= \frac{A_0}{\pi}\int_{-\infty}^{+\infty} dx \int_{\pi}^{\pi} dy \, \exp\left(-\frac{[x+\Delta\delta \sin y]^2}{2\sigma_u^2}\right)\cos n(y+Bx), \end{aligned} \tag{7}$$

where in the last equality we used the property [2]

$$f(y+T) = f(y) \Longrightarrow \int_a^{T+a} dy f(y) = \int_0^T dy f(y), \qquad a \in \mathbb{R}. \tag{8}$$

With the change of variable $t = x + \Delta\delta \sin y$, $dx = dt$ it follows

$$\begin{aligned} a_n &= \frac{A_0}{\pi}\int_{\pi}^{\pi} dy \int_{-\infty}^{+\infty} dt \, e^{-\frac{t^2}{2\sigma_u^2}} \cos n(y + Bt - B\Delta\delta \sin y) \\ &= \frac{A_0}{\pi}\int_{\pi}^{\pi} \cos n(y - B\Delta\delta \sin y) \int_{-\infty}^{+\infty} dt \, e^{-\frac{t^2}{2\sigma_u^2}} \cos nBt, \end{aligned} \tag{9}$$

where in the last equality we used $\int_{-\infty}^{+\infty} dt \, \exp(-t^2/(2\sigma_u^2)) \sin nBt = 0$.

[2] Let $f : \mathbb{R} \to \mathbb{R}$ be continuous and periodic with period T$\in \mathbb{R}^+$, then $\int_a^{a+T} dy f(y) = \int_0^T dy f(y)$ for $a \in \mathbb{R}$. To prove it, define $g(a) := \int_a^{a+T} dy f(y)$. Then g is differentiable and $g'(a) = f(a+T) - f(a) = 0$, thus g is constant.

Proof of the Energy-to-Density Modulation Conversion Formula

For $n = 0$ it follows that

$$a_0 = 2A_0 \int_{-\infty}^{+\infty} dt\, e^{-\frac{t^2}{2\sigma_u^2}} = CI_0. \tag{10}$$

For $n \geq 1$, from the definition of the Bessel function of nth order

$$J_n(x) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} dy\, \cos(ny - x \sin y), \tag{11}$$

it follows that

$$\begin{aligned} a_n &= 2A_0 J_n(nB\Delta\delta) \int_{-\infty}^{+\infty} dt\, e^{-\frac{t^2}{2\sigma_u^2}} \cos nBt = \frac{2A_0}{B} J_n(nB\Delta\delta) \int_{-\infty}^{+\infty} d\tau\, e^{-\frac{\tau^2}{2(nB\sigma_u)^2}} \\ &= 2CI_0 J_n(nCkR_{56}\Delta\delta) e^{-\frac{1}{2}n^2C^2k^2R_{56}^2\sigma_u^2}, \end{aligned} \tag{12}$$

where we used the change of variable $\tau = nBt$ and

$$\int_{-\infty}^{+\infty} dx\, e^{-ax^2} \cos x = \frac{1}{2}\sqrt{\frac{\pi}{a}} e^{-\frac{1}{4a}}, \qquad a > 0. \tag{13}$$

Therefore the final expression for the beam current I read

$$I(z) = CI_0 \left[1 + 2 \sum_{n=1}^{\infty} J_n(nCkR_{56}\Delta\delta) e^{-\frac{1}{2}n^2C^2k^2R_{56}^2\sigma_u^2} \cos(nCkz) \right]. \tag{14}$$