Intra-beam scattering and the ultimate seeding wavelength in EEHG

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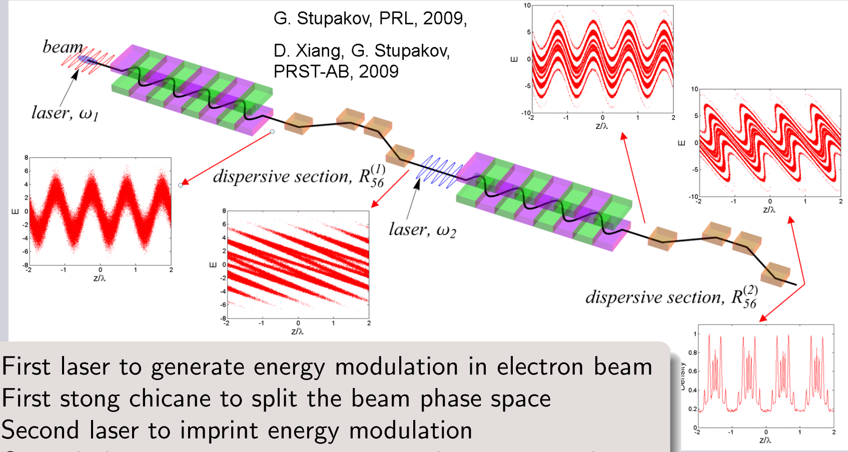
Echo-enabled harmonic generation

EEHG uses a strong dispersion element in the first modulator and adds one more modulator-chicane (radiator-undulator is not shown).



- First laser to generate energy modulation in electron beam
- First stong chicane to split the beam phase space
- Second laser to imprint energy modulation
- Second chicane to convert energy modulation into density modulation

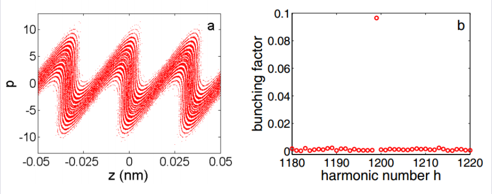
Maximal harmonic number for EEHG?

HGHG up-shifting of the laser frequency is limited by the energy spread of the beam and is typically $\sim 6 - 10$. If EEHG promises a higher frequency multiplication, then what is the maximal harmonic number that can be obtained with EEHG? On paper it can be as large as $\sim 10^3$.

MOPC79 Proceedings of FEL2009, Liverpool, UK

FEASIBILITY STUDY FOR A SEEDED HARD X-RAY SOURCE BASED ON A TWO-STAGE ECHO-ENABLED HARMONIC GENERATION FEL*

D. Xiang†, Z. Huang, D. Ratner and G. Stupakov, SLAC, Menlo Park, CA 94025 USA



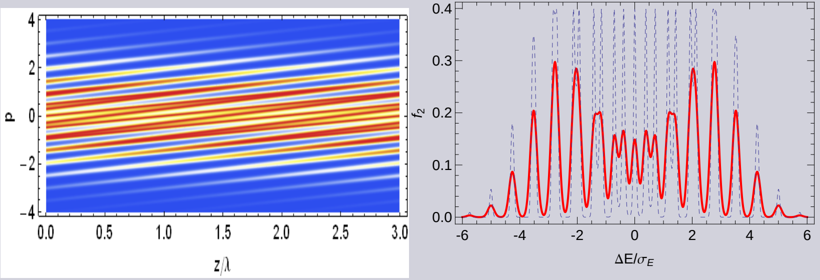
Physics issues with EEHG

In reality there are factors that can limit the ultimate harmonic number (see references at `http://www.slac.stanford.edu/~stupakov/eehg.shtml`):

- Lattice nonlinearities and emittance effects—simulations with `elegant`
- Tolerances on magnetic field in the seed system
- Amplitude and phase control of the seed laser
- Control of the beam parameters (energy, energy chirp, etc.)
- Energy diffusion due to incoherent synchrotron radiation (ISR) in chicanes
- Intra-beam scattering (IBS)

Conceptually important are the last two issues.

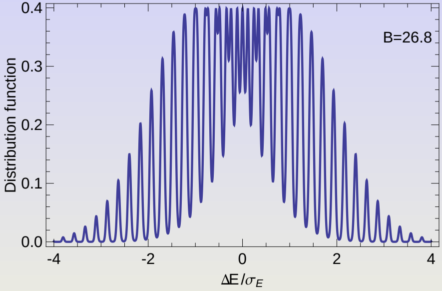
Why ISR and IBS are important?



Large value of $R_{56}^{(1)}$ in the first (strong) chicane generates a fine structure in the energy distribution. The dominant process in ISR and IBS is the diffusion in the momentum space. The diffusion time $\propto (\delta E)^2$. It can smear out the energy modulation and result in the diminished bunching factor at the final harmonic.

Incoherent synchrotron radiation in the first chicane

For the nominal FERMI parameters ($E_0 = 1.2$ GeV, $\sigma_E = 150$ keV) and $\lambda_r = 10$ nm with EEHG the width of the modulation is $\sim 0.2\sigma_E \sim 30$ KeV. The scaling is $\Delta E \sim \sigma_E / h$.



The incoherent energy spread after passing a dipole

$$\Delta\sigma_E = 6.4 \text{ KeV} \times \sqrt{\frac{L\,(\text{m})}{[\rho\,(\text{m})]^3}}\,[E\,(\text{GeV})]^{7/2}$$

can be a fraction of keV. Choosing a larger bending radius (and longer bends) in the chicane would decrease $\Delta\sigma_E$.

IBS

Account of IBS effect adds a collision term to the Vlasov equation. It is derived using the following (see details in Stupakov, FEL 2011):

- Due to the fast modulation of the distribution function over energy in the regions of interest, the dominant term (now in the lab frame) is diffusion over energy

$$\text{coll. term} = \frac{1}{2} D \frac{\partial^2 f}{\partial \Delta E^2}$$

  This term should be added to the RHS of the Vlasov equation.
- The energy spread of a group of initially monoenergetic particles in the beam increases with the distance s

$$\frac{d}{ds}\langle \Delta E^2 \rangle = D$$

  (D has dimension of keV^2/m)

IBS

- *In the beam frame* the distribution function is characterized by the transverse temperatures T_x, T_y and $T_\parallel$, $T_\perp = m\gamma^2\sigma_\theta^2 c^2$, $T_\parallel = mc^2\sigma_\eta^2$. Usually $T_\parallel \ll T_x, T_y$ ($T_x \sim T_y \sim 70$ eV, $T_\parallel \approx 2$ meV assuming $\beta = 20$ m, $E = 2$ GeV and $\sigma_E = 150$ keV). These temperatures are non-relativistic: $T_\parallel, T_\perp \ll mc^2$.
- The diffusion coefficient actually varies in the phase space, but we average it over x, y, θ_x, θ_y and it becomes the function of the slice in the beam, z, and the location in the lattice s: $D(z,s)$. Because of this averaging, D can be calculated neglecting the microbunching due to the energy modulations of the beam.

Diffusion coefficient

Analytical result for D

$$D(z,s) = \frac{\pi^{1/2}\Lambda}{2\gamma\sqrt{\sigma_{\theta x}(s)\sigma_{\theta y}(s)}} \frac{(m_e c^2)^2 r_e}{\sigma_x(s)\sigma_y(s)} \frac{I(z)}{I_A}$$

Λ – Coulomb logarithm
$I_A = 17$ kA
Relatively weak dependence on γ

$$D \sim \frac{I}{\epsilon_N \sigma_\perp} \sim \frac{I\sqrt{\gamma}}{\epsilon_N^{3/2}\sqrt{\beta}}$$

In practical units, assuming $\Lambda \approx 8$

$$D = 3.1 \frac{I\ [\mathrm{kA}]}{(\varepsilon_x\ [\mu\mathrm{m}])(\sigma_x\ [100\ \mu])} \frac{\mathrm{keV}^2}{\mathrm{m}}.$$

Effect of IBS on EEHG—a simple model

In a simple model we assume that collisions occur in a drift section of length ℓ separating the first chicane from the second modulator—in this region the distribution function, being modulated in energy, is most sensitive to the collisions.

Calculation of the bunching factor b_h at h-th harmonic gives

$$b_h = b_h^{(0)} e^{-\ell/L},$$

where $b_h^{(0)}$ is the bunching factor without collisions and

$$L = \frac{2E^2}{Dh^2 (R_{56}^{(2)} k_{\mathrm{L}})^2}$$

Note h^{-2} scaling of L.

Numerical examples

Consider a soft x-ray EEHG FEL scheme with emittance $\epsilon = 1$ µm, beam peak current of 1 kA, and the rms energy spread 100 keV. We also assume the rms transverse bunch size of 100 µm. This gives $D = 3.1$ keV^2/m.
Three EEHG scenarios with the harmonic number $m = 50$, 100 and 200. For all 3 cases we assumed that the dimensionless modulation amplitude were $\Delta E_1/E_0 = 3$ and $\Delta E_2/E_0 = 6$. The bunching factors without collisions, and the decay distance L are shown in Table below.

h	$\Delta E_1/E_0$	$\Delta E_2/E_0$	$b_h^{(0)}$	L (m)
50	3	6	0.088	80
100	3	6	0.071	22.5
200	3	6	0.047	5.6

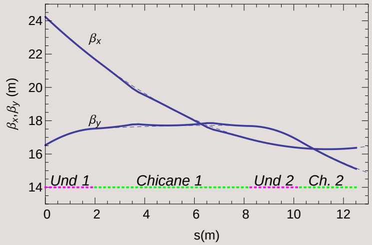
"Practical" example of NGLS EEHG seeding

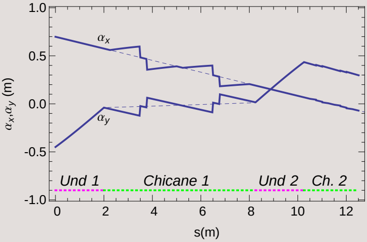
Working parameters for NGLS (parameters and lattice are provided by G. Penn)

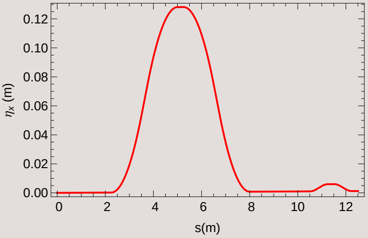
Electron beam energy	2.4 GeV
Bunch peak current	600 A
Normalized emittance	0.6 μm
Energy spread, σ_E	150 keV
Laser wavelength	200 nm
First/second energy modulation	0.5/1 MeV
Seed wavelength	1 nm

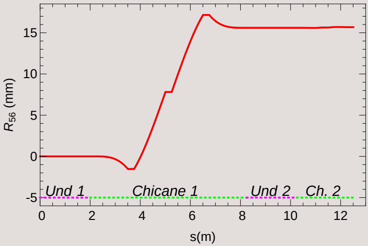
For these parameters, ideally, the bunching factor $b_{200} \approx 0.05$.

Lattice functions, dispersion and R_{56}



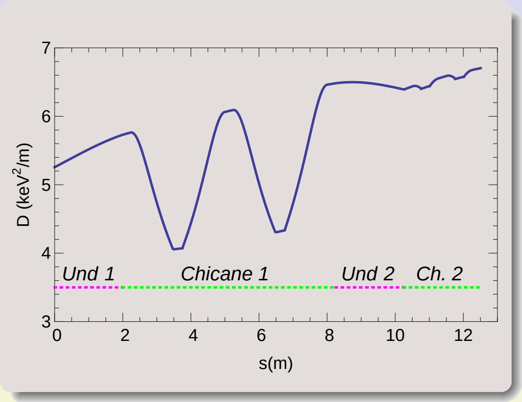






IBS Diffusion coefficient for peak current

Assuming $\Lambda = 8$



With $D \sim 5$ keV^2/m, in 10 m an initially monoenergetic beam would get the rms energy spread of $\sqrt{50} \approx 7$ keV.

Challenge: Vlasov equation with diffusion

Challenge: the energy distribution of the beam changes dramatically through the seeding system. Numerically solving the Vlasov equation with diffusion is not an easy task. Fortunately, there is a new convenient technique, due to N. Yampolsky and B. Carlsten, that makes the problem solvable analytically.

Description of modulated beam dynamics

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(Dated: December 5, 2011)

Beam debunching due to ISR-induced energy diffusion

Nikolai A. Yampolsky and Bruce E. Carlsten
Los Alamos National Laboratory, Los Alamos, New Mexico, 87545, USA

Beam distribution function

The technique works for linear beam dynamics with no collective effects (wakefields).
The standard approach without IBS: the distribution function f

$$f(\Delta E, s)$$

satisfies the Vlasov equation. Let $\vec{X} = (\Delta E, s)^T$ is the column vector. If we know the 2×2 R-matrix from $s = 0$ to s

$$\vec{X}(s) = R \cdot \vec{X}(0)$$

The distribution function $f(\vec{X}, s)$ is easily expressed through $f_0(\vec{X}) = f(\vec{X}, 0)$

$$f(\vec{X}, s) = f_0(R^{-1}\vec{X})$$

Fourier transformation of the distribution function

Make 2-dimensional Fourier transformation of f

$$\hat{f}(k_z, k_{\Delta E}, s) = \int dz d\Delta E e^{i(zk_z + \Delta E k_{\Delta E})} f(\Delta E, s)$$

$\hat{f}$ also satisfies a (transformed) Vlasov equation. Let $\vec{K} = (k_z, k_{\Delta E})^T$ is the column vector, then the distribution function $\hat{f}(\vec{K}, s)$ is easily expressed through $\hat{f}_0(\vec{K}) = \hat{f}(\vec{K}, 0)$ via the transposed matrix, R^T

$$\hat{f}(\vec{K}, s) = \hat{f}_0(R^T \vec{K})$$

Interpretation of the Fourier phase space

We can now think about the 2 numbers $(k_z, k_{\Delta E})$ as a *quasi-particle* in a 2-dimensional Fourier phase space. When the beam is moving through a lattice, quasi-particles are moving in Fourier phase space and carry the value of $\hat{f}$ with them.

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Note that $\hat{f}(k_z, 0, s)$ is proportional to 1D Fourier of the current in the beam. If the distribution function f is normalized by unity, then

$$|b(k_z)| = |\hat{f}(k_z, 0, s)|$$

is the bunching factor ($|b(k_z)|^2$ determines the intensity of coherent radiation at frequency $\omega = ck_z$).

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In analogy with other types of quasi-particles in quantum physics (phonons, plasmons, magnons, etc.) we can call these quasi-particles *bunchions*.

Fourier transformation and diffusion

Why is $\hat{f}$ good? It easily solves the diffusion problem:

$$\frac{1}{2} D \frac{\partial^2 f}{\partial \Delta E^2} \rightarrow -\frac{1}{2} D k_{\Delta E}^2 \hat{f},$$

the differential equation is replaced by an algebraic one.
To compute the effect of diffusion

$$\hat{f}_{\mathrm{diff}} = e^{-(1/2)\int_0^s D(s) k_{\Delta E}(s)^2 ds} \hat{f}_{\mathrm{no\ diffusion}}$$

with $k_{\Delta E}(s)$ the trajectory of the bunchion from initial to final state and $D(s)$ is the diffusion coefficient along the path.

2D phase space

In the longitudinal 2D Fourier phase space, $\vec{K} = (k_z, k_{\Delta E})^T$

$$R = \begin{pmatrix} 1 & R_{56} \\ 0 & 1 \end{pmatrix}$$

and

$$\begin{pmatrix} k_z(s) \\ k_{\Delta E}(s) \end{pmatrix} = (R^T)^{-1} = \begin{pmatrix} 1 & 0 \\ -R_{56}(s) & 1 \end{pmatrix} \begin{pmatrix} k_z(0) \\ k_{\Delta E}(0) \end{pmatrix}$$

or

$k_{\Delta E}(s) = -R_{56}(s) k_z(0) + k_{\Delta E}(0)$

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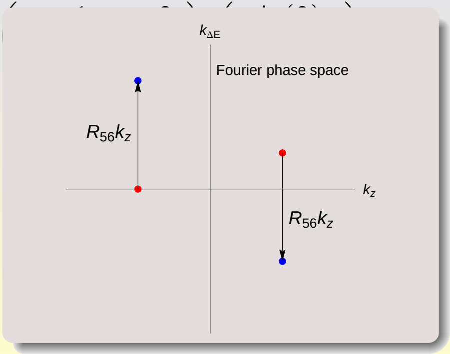
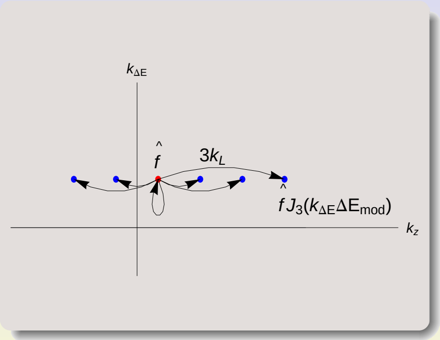


or

$$k_{\Delta E}(s) = -R_{56}(s)k_z(0) + k_{\Delta E}(0)$$

Energy modulation in the Fourier phase space

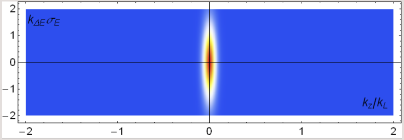
What does the energy modulation in the undulator do to $\hat{f}$?
Scattering and creation of new quasi-particles.



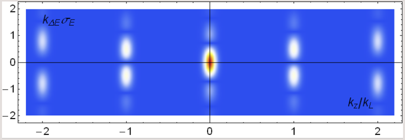
It splits a point $(k_z, k_{\Delta E})$ into many points $(k_z + k_L m, k_{\Delta E})$ shifted horizontally, $m = 0, \pm 1, \pm 2, \ldots$ (k_L is the laser wavenumber) and multiplies $\hat{f}$ at the new location by $J_m(k_{\Delta E}\Delta E_{\mathrm{mod}})$.

Fourier phase space evolution in EEHG

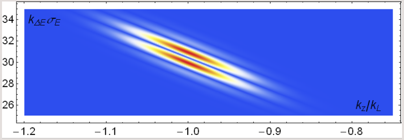
Initial $|\hat{f}|$



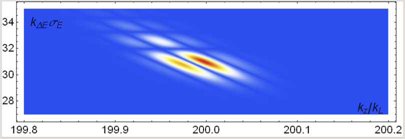
After the first modulator



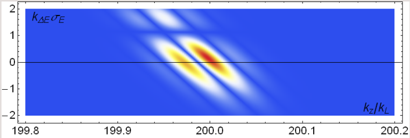
After the first chicane



After the second modulator

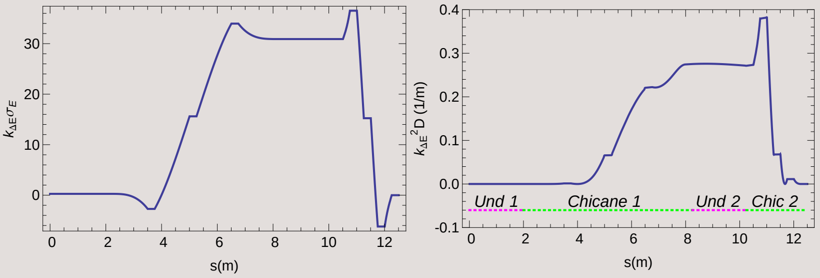


After the second chicane



With $\sigma_E = 150$ keV we have $1/k_{\Delta E} \approx 5$ keV. [The bunch is assumed unrealistically short, $\sigma_z \approx 3\lambda_L$.]

Diffusion in the phase space



The suppression factor

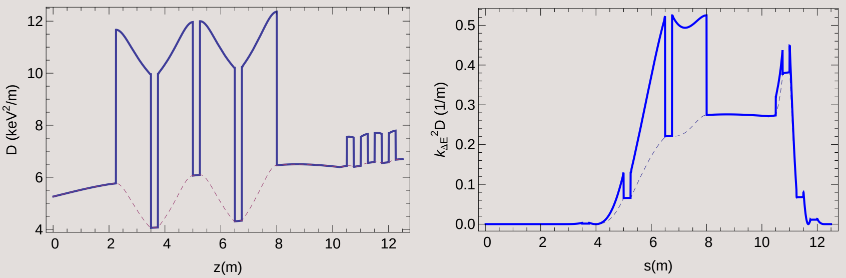
$$e^{-(1/2)\int_0^s D(s) k_{\Delta E}(s)^2 ds} = 0.47$$

Hence the expected bunching factor due to IBS suppression is 2.5%.

Adding ISR to IBS

$$D_{\mathrm{ISR}} = \frac{55}{24\sqrt{3}} \frac{\hbar e^2 c}{\rho^3} \gamma^7$$

The sum of D_{ISR} and D_{IBS}



The suppression factor is 0.35, the bunching $b_{200} = 0.017$. As one can see from these results, the Coulomb collisions represent a serious limiting factor for the EEHG seeding in the range of harmonic numbers exceeding 10^2.

Summary

While there are practical limitations for the maximally achievable harmonic multiplication in EEHG, IBS and ISR set a *conceptual limit* on h. It is due to the small energy spread in the structures created by the first chicane in the phase space of the beam. A new technique for calculation of the effect of the energy diffusion on evolution of the beam distribution function was presented which uses the Fourier transformation over the phase space variables. It allows to analytically account for the IBS and ISR in EEHG. My estimate of the maximal attainable h due to these effects would be in the range 100-200.